SYLLABUS

1. **Inviscid incompressible flows:**

   1.1 Fluids: general properties, characterization and kinematics. Continuum hypothesis. Density.

   1.2 Material (Lagrangian) derivative.

   1.3 Pathlines, streamlines. Steady flows.

   1.4 Volume and mass fluxes. Incompressible fluids.

   1.5 Ideal fluids (I): surface stresses and volume forces.

   1.6 Mass conservation.

   1.7 Momentum balance equation for inviscid fluids (Euler).

   1.8 Ideal fluids (II): Bernouilli's Theorem. Vorticity.

   1.9 Vorticity equation. 2D case: streamfunction. Mass flow.

   1.10 Steady two-dimensional incompressible and irrotational inviscid flows: complex potential.

   1.11 Circulation. Kelvin's Theorem.

2. **Viscous flows:**

   2.1 Viscous fluids: viscosity, shear stress and Newtonian hypothesis.

   2.2 Navier-Stokes equations. No-slip boundary conditions. Stress-free boundary conditions.

   2.3 Dimensional analysis. Reynolds number. Viscous and dynamic time.

   2.4 Canonical flows (I) (steady-cartesian): plane Couette-Poiseuille (two-dimensional case: streamfunction formalism).

   2.5 Canonical flows (II) (steady-cylindrical): Hagen-Poiseuille flow, Taylor-Couette flow, spiral Poiseuille-Couette flows.

   2.6 Canonical flows (III): unsteady cartesian (Stokes problems)

   2.7 Self-similar flows: boundary layers Prandtl theory

3. **Thermal buoyancy, rotation effects and geophysical flows:**

   3.1 Flows in rotating frames. Coriolis force.

   3.2 Energy balance equation. Boundary conditions.

   3.3 Boussinesq approximation and buoyancy.

   3.4 Geophysical flows. Reynolds-averaged equations and turbulent mixing.

   3.5 Hydrostatic balance. Shallow water model.

   3.6 Barotropic waves: Kelvin waves and Poincaré waves. Applications: tides and tsunamis.

4. **Hydrodynamic stability theory, transition to turbulence and deterministic chaos:**

   4.1 Hydrodynamic stability: motivation, phenomenology and history.

   4.2 Systems of nonlinear differential equations.

   4.3 Invariant sets: equilibria and limit cycles.

   4.4 Linear stability of equilibria. Hartman-Grossmann Theorem. Hyperbolic manifolds.

4.5 Parameter-dependent systems. Topological equivalence. Local bifurcation (definition).

4.6 Classical bifurcation scenarios: saddle-node, pitchfork and Hopf bifurcations. Examples.

4.7 Linear stability analysis of Navier-Stokes flows. Absolute and convective instabilities.

4.8 Applications and models: Lorenz, Eckhaus, Rayleigh-Benard (stress-free case)

5. Instabilities in parallel shear flows:

5.1 Stability of parallel shear flows. Orr-Sommerfeld equation.

5.2 Applications: linear stability of plane Poiseuille flow.

6. Instabilities in centrifugal flows:

6.1 Rayleigh criterion of inviscid stability.

6.2 Applications: linear stability of Taylor-Couette flow.

7. Instabilities due to thermal buoyancy:

7.1 Stability of thermal convection flows.

7.2 Applications: linear stability of Rayleigh-Bénard problem.

8. Modelling geophysical flows: long waves in shallow waters:

8.1 Modelling long waves in shallow waters.

8.2 Application: modelling tides and tsunamis.

Assessment and final grade:

- 50 % Mid-term exam on fundamentals of fluid mechanics.
- 30 % Assignments based on practicals of Topics 5 to 8.
- 20 % Extra assignment and oral presentation.

Bibliography:

Fundamentals of fluid dynamics:

- Acheson, D. J., *Elementary fluid dynamics*, Oxford University Press, 1990. ISBN 0198596790.
- Chorin, A., Marsden J. A. *A mathematical introduction to fluid mechanics*, 3rd ed., Springer-Verlag, 1992. ISBN B10834722.
- Kundu, P. K., *Fluid mechanics*, 6th ed., Academic Press, 2015. ISBN 978-0124059351.
- Paterson, A. R., *A First course in fluid dynamics*, Cambridge University Press, 1983. ISBN 0521274249.

Fundamentals of geophysical flows:

- Cushman-Roisin, B.; Beckers, J. M., *Introduction to geophysical fluid dynamics: physical and numerical aspects*, 2nd ed. Waltham, Elsevier Academic Press, 2011. ISBN 9780120887590.

Hydrodynamic stability and nonlinear dynamics:

- Drazin, P. G., *Introduction to hydrodynamic stability*, Cambridge University Press, 2002. ISBN 0521009650.
- Kuznetsov, Y. A., *Elements of applied bifurcation theory*, 3rd ed., Springer, 2004. ISBN 0387219064.
- Wiggins, S. *Introduction to applied nonlinear dynamical systems and chaos*, 2nd ed., Springer-Verlag, 2003. ISBN 0387001778.