RELATIVIDAD GENERAL (Q2)

Program of General Relativity.

1. *Complements of tensor algebra and differential geometry.*
    a. Basic concepts on tensor algebra.
    b. Tensor fields. Operations.
    c. Vector fields, differential forms and tensor fields in R^n.
    d. Differentiable manifolds.
    e. Differentiable operators. Derivations.
    f. Covariant derivative. Christoffel symbols. Connections.
    g. Parallel transport. Geodesics. Equations.
    h. Metric tensor. Symmetries and Killing vectors.
    i. Vector fields and their invariant classification: Acceleration, Expansion, Shear, and Rotation.
    j. Torsion tensor of a connection. Levi-Civita connection.
    k. Curvature tensors (Riemann, Ricci). Properties.
    l. Submanifolds: Hypersurfaces, 2-surfaces, World-lines.
    m. First and second fundamental forms.
2. *Review on Special Relativity: Minkowskian formulation of the Special Relativity.*
    a. Postulates of the Special Relativity.
    b. Minkowski's metrics and Minkowskian space-time. Inertial observers.
    c. Four-vectors. Light cone. Lorentzian geometry.
    d. Lorentz and Poincaré transformations and groups.
    e. Relativistic kinematics and relativistic dynamics.
    f. The electromagnetic tensor: Maxwell equations.
3. *Principles of General Relativity.*
    a. The principle of General Relativity.
    b. Mach's Principle.
    c. Inertial and gravitational mass. The lift experiments.
    d. The principle of equivalence.
    e. The problem of the rotating disc. Non-Euclidean geometries.
    f. The principle of general covariance.
    g. The correspondence principle.
4. *The equations of General Relativity.*
    a. Foundations of General Relativity. Newton's equations of gravitation.
    b. Consequences of the Equivalence Principle.
    c. Postulates and principles of General Relativity.
    d. Stress-Energy-Momentum tensor. Conservation of the SEM tensor.
    e. Einstein tensor. Properties.
    f. Equations of GR: Einstein field equations and geodesic equation.
    g. Consequences and properties of Einstein field equations. Tidal effects. The cosmological constant.

   h. General relativity from a variational principle: the Hilbert-Einstein Lagrangian.

5. *General relativity kinematics.*
   a. Distances and time intervals I general relativity. Three-dimensional metric tensor.
   b. The synchronization criteria of clocks in General relativity.
   c. Locally-inertial reference frame.
   d. Gravitational red-shift. A covariant generalization of Doppler and gravitational red-shift.
6. *The Schwarzschild solution. Geodesics in Scwarzschild geometry.*
   a. Spherically-symmetric solutions. Static solutions. Asymptotically flat solutions.
   b. The Schwarzschild spacetime. Properties and the Birkhoff theorem.
   c. Singularities of the curvature (essential, intrinsic o real) and removable singularities (singularities of the coordinates).
   d. Lagrangian method to obtain the equation of motion of timelike (or null) geodesics.
   e. Symmetries and conserved quantities.
   f. Bound orbits. Stable and unstable circular orbits. Radial geodesics.
7. *Experimental tests of General Relativity.*
   a. Advance of the perihelion of Mercury.
   b. Deflection of light rays.
   c. Gravitational Red-Shift.
   d. Radar time delay.
8. *Space-time diagrams in Schwarzschild coordinates.*
   a. Eddington-Finkelstein coordinates.
   b. Event horizon.
   c. Black Holes. A classical argument.
   d. Tidal forces in a black hole.
   e. Observational evidence for black holes.
9. *Maximal extension and conformal compactification.*
   a. Maximal analytic extensions
   b. The Kruskal solution
   c. Penrose diagram for a Minkowski space-time.
   d. Penrose diagram for a Kruskal solution.
   e. Black Holes versus White Holes. Closed trapped surfaces.
   f. Spherically symmetric gravitational collapse of a star. Creation of black holes.
10. *The Vaidya metric.*
   a. External spacetime of a spherically symmetric and nonrotating star which is either emitting or absorbing null dust: Vaidya metric.
   b. Flux of radiation. Stress Energy Tensor.
   c. Radiating black holes. Penrose diagrams of Vaidya metric.
   d. Radiating collapse of a spherically symmetric space-time: Matching conditions.
   e. Dominant energy conditions in a radiative collapse.
11. *Relativistic Cosmology. Cosmological models.*
   a. Olbers' Paradox.
   b. Hubble's Law.

c. The cosmological principle. Weyl’s postulate.
d. Relativistic cosmology.
e. The geometry of 3-spaces of constant curvature.
f. Friedmann’s equations.
g. A cosmological definition of distance.
h. Hubble’s law in relativistic cosmology.
i. The flat space models.
j. Friedmann-Lemaître-Robertson-Walker models, in a flat case. Big-Bang.
k. Conformal structure of a “flat” Friedmann-Lemaître-Robertson-Walker model.

Bibliography:

- Ray D’Inverno: “Introducing Einstein’s Relativity”. Clarendon Press (1998).
- L.D. Landau and E.M. Lifshitz: “The classical theory of fields”. Butterworth and Heinemann. (2007).
- Ch. Misner, K. Thorne and J.A. Wheeler: “Gravitation”. W. H. Freeman (1973).
- Sean M. Carroll: “Spacetime and Geometry: an introduction to General Relativity”. Addison-Wesley (2004).
- James J. Callahan: “The Geometry of Spacetime”. Springer (2001).
- James B. Hartle: “Gravity: An Introduction to Einstein’s General Relativity”. Addison- Wesley, (2003).
- Bernard Schutz: “Gravity: From the ground up”. Cambridge Univ. Press (2007).

Invited Conference:

Evaporation of Black Holes, by Prof. Ramón Torres Herrera (FA).

Contenidos, forma de trabajo y método de evaluación:

Los temas 1, 2, 3 y 4 del programa se desarrollarán de la manera habitual (clase estándar) y ocuparán las 30 primeras horas lectivas del curso (aproximadamente).

Una vez concluida esta parte, habrá una prueba de examen (parcial) para valorar el grado de asimilación de los contenidos (E1).

Los 7 temas restantes se impartirán en forma de 5 conferencias, agrupados de la siguiente manera:

i. Cinemática en Relatividad General,
ii. Solución de Scwarzschild (Trayectorias Geodésicas, Extensión máxima y compactificación conforme),
iii. Las pruebas experimentales de la Relatividad General,
iv. La métrica de Vaidya,
v. Cosmología relativista.

Cada conferencia tendría una duración aproximada de 2h, con una exposición (no demostrativa) del contenido, remarcando los principales resultados relativos a cada uno de los temas y la novedad que significaron en el contexto de la mentalidad newtoniana clásica, y los desarrollos que han tenido hasta el presente.

Se implementarán diversos ejercicios y/o pequeños trabajos sobre estos contenidos para que los estudiantes los desarrollen y los entreguen para su valoración (E2).

Al finalizar el curso se realizará un examen final opcional sobre el total de los contenidos de la asignatura (E3).

La nota de la asignatura se obtendrá de la media (E1+E2)/2, o bien directamente de E3.

Habrá una ***conferencia invitada***: *Evaporation of Black Holes*, por el Prof. *Ramón Torres Herrera* (FA), que se impartirá después de la conferencia (ii).