Program of General Relativity.

1. ***Complements of tensor algebra and differential geometry.***
   a. Basic concepts on tensor algebra.
   b. Tensor fields. Operations.
   c. Vector fields, differential forms and tensor fields in R^n.
   d. Differentiable manifolds.
   e. Differentiable operators. Derivations.
   f. Covariant derivative. Christoffel symbols. Connections.
   g. Parallel transport. Geodesics. Equations.
   h. Metric tensor. Symmetries and Killing vectors.
   i. Torsion tensor of a connection. Levi-Civita connection.
   j. Curvature tensors (Riemann, Ricci). Properties.
2. ***Review on Special Relativity: Minkowskian formulation of the Special Relativity.***
   a. Postulates of the Special Relativity.
   b. Minkowski's metrics and Minkowskian space-time. Inertial observers.
   c. Four-vectors. Light cone. Lorentzian geometry.
   d. Lorentz and Poincaré transformations and groups.
   e. Relativistic kinematics and relativistic dynamics.
   f. The electromagnetic tensor: Maxwell equations.
3. ***Principles of General Relativity (GR).***
   a. Foundations of General Relativity. Newton's equations of gravitation.
   b. Inertial and gravitational mass.
   c. The Principle of Equivalence. Consequences.
   d. Non-Euclidean geometries.
   e. The postulates of General Relativity
4. ***The equations of General Relativity.***
   a. Stress-Energy-Momentum tensor.
   b. Einstein tensor. Properties.
   c. Equations of GR: Einstein field equations and geodesic equation.
   d. Consequences and properties of Einstein field equations. Tidal effects. The cosmological constant.
   e. General relativity from a variational principle: the Hilbert-Einstein and the Einstein-Palatini Lagrangians.
   f. Phenomenological aspects.
5. ***Kinematics in General Relativity.***
   a. Time-like geodesic congruencies.
   b. Distances and time intervals in General Relativity. Three-dimensional metric tensor.
   c. The synchronization criteria of clocks in General relativity.
   d. Locally-inertial reference frame.
   e. Gravitational red-shift. A covariant generalization of Doppler and gravitational red-shift.
6. ***The Schwarzschild solution.***
   a. Spherically-symmetric solutions. Static solutions. Asymptotically flat solutions.

b. The Schwarzschild spacetime. Properties and the Birkhoff theorem.
c. Singularities of the curvature (essential, intrinsic o real) and removable singularities (singularities of the coordinates).
d. Lagrangian method to obtain the equation of timelike (or null) geodesics.
e. Symmetries and conserved quantities.
f. Bound orbits. Stable and unstable circular orbits. Radial geodesics.

7. ***Experimental tests of General Relativity.***
   a. Advance of the perihelion of Mercury.
   b. Deflection of light rays.
   c. Gravitational Red-Shift.
   d. Electromagnetic waves time delay.
8. ***Black holes.***
   a. Eddington-Finkelstein coordinates.
   b. Event horizon.
   c. Black Holes. A classical argument.
   d. Tidal forces in a black hole.
   e. Observational evidence for black holes.
9. ***Maximal extension and conformal compactification.***
   a. Maximal analytic extensions.
   b. The Kruskal solution
   c. Penrose diagram for a Minkowski space-time.
   d. Penrose diagram for a Kruskal solution.
   e. Black Holes versus White Holes. Closed trapped surfaces.
   f. Gravitational collapse of a star. Creation of black holes.
10. ***Radiation modeling and collapse: Vaidya solution.***
    a. External spacetime of a star that emits or receives radiation: Vaidya metric.
    b. Flux of radiation. Stress-energy-momentum tensor.
    c. Black holes and radiation. Penrose diagrams of Vaidya metric.
    d. Radiating collapse of a spherically symmetric space-time: Matching conditions.
    e. Dominant energy conditions in a radiative collapse.
11. ***Relativistic Cosmology. Cosmological models.***
    a. Olbers' Paradox. Hubble's Law.
    b. The cosmological principle. Weyl's postulate.
    c. Friedmann equations.
    d. Relativistic cosmology.
    e. The geometry of 3-spaces of constant curvature.
    f. The flat space models.
    g. Friedmann-Lemaître-Robertson-Walker models in a flat case. Big-Bang.
    h. Conformal structure of a "flat" Friedmann-Lemaître-Robertson-Walker model.
    i. Inflation, dark matter and dark energy problems.

Bibliography:

- S.M. Carroll: "Spacetime and Geometry: an introduction to General Relativity". Addison-Wesley (2004).
- R. D'Inverno: "Introducing Einstein's Relativity". Clarendon Press (1998).

- L.D. Landau and E.M. Lifshitz: “The classical theory of fields”. Butterworth and Heinemann. (2007).
- C. Misner, K. Thorne, and J.A. Wheeler: “Gravitation”. W. H. Freeman (1973).
- E.A. Poisson, “Relativist’s toolkit: the mathematics of black-hole mechanics”. Cambridge, UK ; New York: Cambridge University Press, 2004. ISBN 0521830915, 9780511606601.