Program of General Relativity.

1. *Complements of tensor algebra and differential geometry.*
   a. Basic concepts on tensor algebra.
   b. Tensor fields. Operations.
   c. Vector fields, differential forms and tensor fields in R^n.
   d. Differentiable manifolds.
   e. Differentiable operators. Derivations.
   f. Covariant derivative. Christoffel symbols. Connections.
   g. Parallel transport. Geodesic Equation.
   h. Metric tensor. Symmetries and Killing vectors.
   i. Torsion tensor of a connection. Levi-Civita connection.
   j. Curvature tensors (Riemann, Ricci). Properties.
   k. Submanifolds: Hypersurfaces, 2-surfaces, World-lines.
2. *Review on Special Relativity: Minkowskian formulation of the Special Relativity.*
   a. Postulates of the Special Relativity.
   b. Minkowski's metrics and Minkowskian space-time. Inertial observers.
   c. Four-vectors. Light cone. Lorentzian geometry.
   d. Lorentz and Poincaré transformations and groups.
   e. Relativistic kinematics and relativistic dynamics.
   f. The electromagnetic tensor: Maxwell equations.
3. *Principles of General Relativity.*
   a. The principle of General Relativity.
   b. Mach's Principle.
   c. Inertial and gravitational mass. The lift experiments.
   d. The principle of equivalence.
   e. The problem of the rotating disc. Non-Euclidean geometries.
   f. The principle of general covariance.
   g. The correspondence principle.
4. *The equations of General Relativity.*
   a. Foundations of General Relativity. Newton's equations of gravitation.
   b. Consequences of the Equivalence Principle.
   c. Postulates and principles of General Relativity.
   d. Stress-Energy-Momentum tensor. Conservation of the SEM tensor. Four canonical forms of the SEM. Perfect fluid as a Type I canonical form of SEM: Conservation laws of the SEM tensor for an observer commoving with the fluid.
   e. Einstein tensor. Properties.
   f. Equations of GR: Einstein field equations and geodesic equation.
   g. Consequences and properties of Einstein field equations. Geodesic deviation. Tidal effects. The cosmological constant.
   h. General relativity from a variational principle: the Hilbert-Einstein Lagrangian.
5. *Kinematics in General Relativity.*
   a. Description of Hypersurfaces: Normal vector field. Projector tensor or transverse metric. The character of coordinate system: Temporal, spatial and null coordinates.
   b. Energy conditions: Weak energy conditions, Dominant energy condition, strong energy condition.
   c. Kinematics of a deformable medium. Congruence of timelike geodesics: Expansion, Shear and Rotation. Frobenius' theorem. Raychaudhuris equation. Focussing theorem. Congruence of null geodesics: Expansion, Shear and Rotation. Frobenius' theorem. Raychaudhuris equation. Focussing theorem.
   d. Distances and time intervals in GR. Three-dimensional metric tensor.
   e. The synchronization criteria of clocks in General relativity.

f. Local flatness. Fermi normal coordinates.
g. Synchronous reference system: Gaussian coordinates.
h. Stationary spacetime: Adapted coordinate.
i. The Newtonian limit.
j. The Geometric Optics in curved spacetimes: Light rays. A covariant generalization of Doppler and Gravitational red-shift. Gravitational red-shift : Weak field case.

6. *The Schwarzschild solution.*
   a. Spherically-symmetric solutions. Static solutions. Asymptotically flat solutions.
   b. The Schwarzschild spacetime. Properties and the Birkhoff theorem.
   c. Singularities of the curvature (essential, intrinsic o real) and removable singularities (singularities of the coordinates).
7. *Geodesics in Scwarzschild geometry.*
   a. Conserved quantities for motion in Schwarzschild geometry.
   b. Orbits for particles. Effective potential.
   c. Orbit of a null mass particles. Impact parameter.
8. *Experimental tests of General Relativity.*
   a. Advance of the perihelion of Mercury.
   b. Deflection of light rays.
   c. Gravitational Red-Shift.
   d. Radar time delay.
9. *Non rotating Black Holes.*
   a. Spacetime diagrarm in Schwarzschild coordinates.
   b. Eddington-Finkelstein coordinates.
   c. Event Horizons (EH) and Apparent Horizons A3H.
   d. Black Holes. A classical argument.
   e. Tidal forces in a black hole.
   f. Observational evidence for black holes.
10. *Maximal extension and conformal compactification.*
    a. Maximal analytic extensions
    b. The Kruskal solution
    c. Penrose diagram for a Minkowski space-time.
    d. Penrose diagram for a Kruskal solution.
    e. Black Holes versus White Holes. Closed trapped surfaces.
    f. Spherically symmetric gravitational collapse of a star. Creation of black holes.
11. *The Vaidya metric.*
    a. External spacetime of a spherically symmetric and nonrotating star which is either emitting or absorbing null dust: Vaidya metric.
    b. Flux of radiation. Stress Energy Tensor.
    c. Radiating black holes. Penrose diagrams of Vaidya metric: Event Horizons (EH) and Apparent Horizons A3H.
    d. Radiating collapse of a spherically symmetric space-time: Matching conditions.
    e. Dominant energy conditions in a radiative collapse.
12. *Relativistic Cosmology. Cosmological models.*
    a. The cosmological principle. Weyl's postulate.
    b. Relativistic cosmology.
    c. Spaces of constant curvature.
    d. Friedmann's equations.
    e. Hubbel's law in relativistic cosmology.

Bibliography:

- James J. Callahan: “The Geometry of Spacetime”. Springer (2001).
- Sean M. Carroll: “Spacetime and Geometry: an introduction to General Relativity”. Addison-Wesley (2004).
- James B. Hartle: “Gravity: An Introduction to Einstein’s General Relativity”. Addison- Wesley (2003).
- Hawking-Ellis. “The large scale structure of spacetime”, Cambridge Monographs on Mathematical Physics, Cambridge (1973).
- Ray D’Inverno: “Introducing Einstein’s Relativity”. Clarendon Press (1998).
- L.D. Landau and E.M. Lifshitz: “The classical theory of fields”. Butterworth and Heinemann (2007).
- Ch. Misner, K. Thorne and J.A. Wheeler: “Gravitation”. W. H. Freeman (1973).
- Eric Poisson, “A relativist’s Toolkit”, Cambridge University Press, Cambridge (2004).
- Bernard Schutz: “Gravity: From the ground up”. Cambridge Univ. Press (2007).

Método de evaluación:

Dos examens parcials (P1 i P2). Examen final (F).

Qualificació: La millor de (P1+P2)/2 i F.