

1. Numerical Linear Algebra.

- 1.1 Gauss Elimination (**GEM**) and **LU** factorization. **PA=LU** factorization with partial pivoting.
- 1.2 Least squares problem: **QR** factorization, orthogonal matrices, Householder reflectors.
- 1.3 Conditioning: matrix norms, condition numbers and sensitivity of linear systems.
- 1.4 Gram-Schmidt orthonormalization: numerical instability of **GS** and Reorthogonalized-**GS**.
- 1.5 Iterative Krylov Methods: Krylov subspaces, **GMRES** and Arnoldi iteration.

2. Systems of Nonlinear Equations.

- 2.1 Newton's method. Jacobian approximation.
- 2.2 Parameter-dependent systems. Homotopy and pseudo-arclength continuation.

3. Numerical Fourier Analysis.

- 3.1 Generalized Fourier Series.
- 3.2 The Discrete Fourier Transform. Fourier Interpolant. Aliasing.
- 3.3 Numerical Differentiation in physical and Fourier spaces.

4. Numerical discretization of Ordinary Differential Equations.**4.1 Boundary value problems**

- 4.1.1 Existence and uniqueness of solutions in bounded domains.
- 4.1.2 Generalized Robin boundary conditions. Differentiation matrix modification.
- 4.1.3 Eigenvalue and nonlinear two-point boundary value problems.
- 4.1.4 Periodic domains: Fourier differentiation matrices (Hill-Mathieu equation).
- 4.1.5 Unbounded domains: domain transformation maps (Schrödinger equation).

4.2 Initial value problems

- 4.2.1 Explicit one-step formulas: Euler and Runge-Kutta methods.
- 4.2.2 Explicit Linear Multistep Formulas: Adams-Bashforth methods.
- 4.2.3 Implicit Linear Multistep Formulas: Curtiss-Hirschfelder methods.
- 4.2.4 Convergence and stability of time-steppers. Dahlquist equivalence and barriers.
- 4.2.5 A-stability of time-steppers, stiffness.

Bibliography

1. G. Dahlquist, A. Björck, *Numerical Methods in Scientific Computing, vols. I and II*, SIAM, 2008.
2. A. Meseguer, *Fundamentals of Numerical Mathematics for Physicists and Engineers*, Wiley, 2020.
3. A. Quarteroni, R. Sacco, F. Saleri, *Numerical Mathematics*, Springer, 2007.
4. D.S. Watkins, *Fundamentals of Matrix Computations, 3rd edition*, Wiley, 2010.

Grading

Course's grade G_c is obtained through the formula

$$G_c = 0.8 \times \max\{0.3 \times G_{mt} + 0.7 \times G_f, G_f\} + 0.2 \times G_{lab},$$

where G_{mt} and G_f are the grades corresponding to the mid-term and final exams, respectively, and G_{lab} is the average grade corresponding to the computational lab reports.